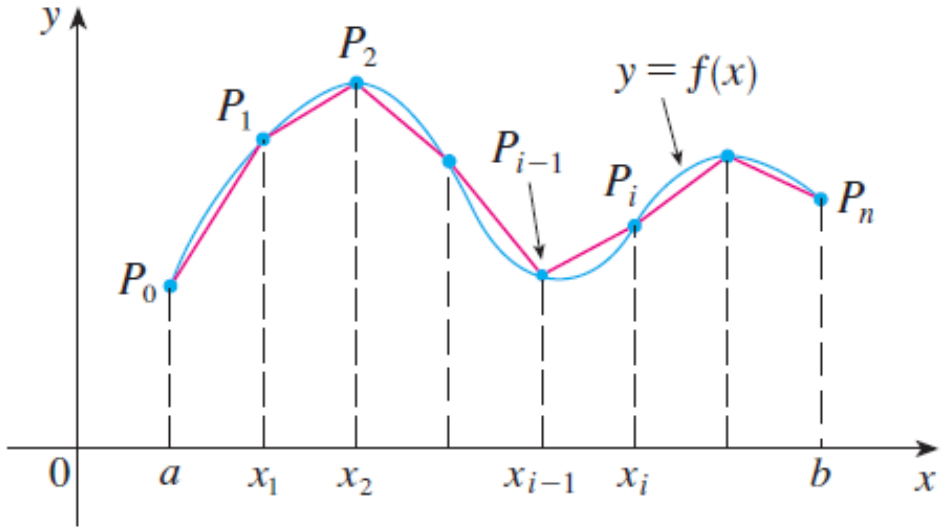
Closing Today: HW_7A, 7B, 7C (7.8, 8.1)
Midterm 2 is Thursday, May 18
Covers: 6.4, 6.5, 7.1-7.5, 7.7, 7.8, 8.1
Today:
Finish Arc Length and Review for Exam 2

8.1 Arc Length (continued)

Last time we derived:

$$\text{Arc Length} = \int_a^b \sqrt{1+(f'(x))^2}\, dx$$



Good news: We have a method to write down an integral for arc length.
Bad news: The arc length integral rarely can be evaluated explicitly. In HW, you see a few, unusual, cases where you can compute arc length.

Entry Task: Two homework questions.
Find the arc length of

$A)\ y = \frac{x^4}{8} + \frac{1}{4x^2}$ for $1 \le x \le 2$.

$B)\ y = \ln(1 - x^2)$ for $0 \le x \le 1/7$.

A Brief Exam 2 Review

Integration!

1. The 4 special methods

By parts: $xe^{3x}, x^2\cos(5x)$, $\frac{\ln(x-1)}{x^2}, x\tan^{-1}(x)$.

Trig: $\sin^3(x)\cos(x), \cos^4(x)$, $\sec^3(2x)\tan^3(2x)$

Trig sub: $\frac{1}{x\sqrt{x^2-9}}, \frac{1}{(4-x^2)^{3/2}}$, $\sqrt{x^2+6x+10}$.

Part Frac: $\frac{x+2}{(x-1)(x-3)}, \frac{4x}{(x-1)^2(x-3)}$, $\frac{5}{x(x^2+4)}, \frac{x^2}{x+7}, \frac{x-3}{x^2+8x+20}$

2. Substitution and Simplifying

Try $u = \sqrt{x}$, u = inside, u = e^x, u = trig.

Know trig facts, such as $\tan(x) = \frac{\sin(x)}{\cos(x)}, \sec(x) = \frac{1}{\cos(x)}$. square identities, and half-angle.

3. Improper Integrals:

a) Rewrite as a limit!!

b) Integrate

c) Take limit

4. Trapezoid/Simpson Rules

a) Set up integral, then compute width and label tickmarks.

b) Use formula.

Random Integrals from Old Finals:

1. $\int \frac{1-x}{\sqrt{1-x^2}}\,dx$
2. $\int \frac{x^2-x+8}{x^3+4x}\,dx$
3. $\int 2x\ln(x+5)\,dx$
4. $\int \cos^3(x)\,dx$
5. $\int_0^2 \frac{1}{\sqrt{x^2+2x+4}}\,dx$
6. $\int_1^3 \frac{1}{x^2+x^3}\,dx$
7. $\int \tan^2 x \sec^4(x)\,dx$
8. $\int \frac{1}{\left(1+\sqrt{x}\right)^3}\,dx$
9. $\int \sin(x)\sqrt{\cos(x)}dx$

Random Improper Integrals:

1. $\int_1^2 \frac{x}{\sqrt{x-1}}\,dx$
2. $\int_{-3}^{\infty} xe^{-x}\,dx$
3. $\int_1^{\infty} \frac{1}{\sqrt{x}(1+x)}\,dx$

Approximation:

1. Use Simpson's Method with n = 4 subdivision to approximate the value of

$$\int_0^4 \sqrt{1+4x^4}\,dx$$

5. New Applications

a) **Average value** = $\frac{1}{b-a}\int_a^b f(x)dx$

b) **Arc Length**

c) **Work** = $\int_a^b (Force)(Dist)$

(i) If f(x) = "force formula at x", then

Force = f(x), *Dist* = Δx;

$$\text{Work} = \int_a^b f(x)dx$$

(ii) *Chain/Cable*: k = force/length

If you label top: x = 0, then

Force = k Δx, *Dist* = x;

$$\text{Work} = \int_a^b k\ \text{x}\ dx$$

(iii) *Pumping*: k = force/volume

If top is y = b, then

Force = k(Area)Δy, *Dist* = b – y;

$$\text{Work} = \int_a^b k(\text{Area})(b-y)dy$$

Applications from old tests:

1. Find the average value of $\cos^3(x)$ on the interval 0 to $\pi/2$.

2. A spring has natural length of 30 cm from the wall. It requires 2 J of work to stretch it from 40 cm to 45cm (from the wall). How far beyond its natural length will a force of 64 N keep the spring stretched?

3. A 1600 lb elevator is suspended by a 200 ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?

4. A rope is used to pull a bucket full of water up from a well that is 10 m deep. The rope has a total mass of 5 kg. The bucket has a mass of 11 kg. Find the total work done in lifting the bucket to the top (Recall: Accel. due to gravity is 9.8 m/s^2)

5. A well is in the shape of a cylinder of radius 1 meter and depth 8 meters. It is half full of water. Find the word to pump all the water to the top. (Recall: Water weighs 9800 N/m^3)

6. The portion of the graph $y = x^2 / 9$ between $x = 0$ and $x = 3$ is rotated about the y-axis to form a container. The container is full of a liquid that has density 100 lbs/ft^3. Find the work required to pump all the liquid to the top of the container.